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not attain to cutting  $LS$  produced in the point  $X$ , situated without the cavity of the present curve, unless previously the prolonged  $FT$  cuts in some point  $O$  the portion of this curve between the points  $S$  and  $K$ .

Hoc autem erat demonstrandum.

COROLLARY. And hence flows manifestly, that between the tangent of this curve and the curve itself cannot be placed any straight [ray], whether on one or the other side of the tangent, which falls wholly without the cavity of the curve ; since a straight [ray] so located must make (from the preceding) an acute angle with the perpendicular let fall from the point of contact to the opposite base.

[To be Continued.]

## DEPARTMENTS.

### SOLUTIONS OF PROBLEMS.

#### ARITHMETIC.

102. Proposed by ALOIS F. KOVARIK, Professor of Mathematics, Decorah Institute, Decorah, Iowa.

A's age is to B's as 2:3. 20 years from now their ages will be to each other as 4:5. What are their ages, respectively ?

I. Solution by J. OWEN MAHONEY, B. E., M. Sc., Master of Mathematics and Science, Carthage Graded and High School, Carthage, Texas.

It is easily seen that A's age : A's age + 20 years :: 2 : 4,  
or A's age : 20 years :: 2 : 4.

∴ A's age = 20 years. But B's age =  $\frac{3}{2}$  of A's = 30 years.

II. Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa.

In the first instance, A's age =  $\frac{2}{3}$  B's age.

In the second instance, A's age =  $\frac{4}{5}$  B's age.

∴  $\frac{2}{3}$  B's age + 20 years =  $\frac{4}{5}$  (B's age + 20 years).

∴  $\frac{2}{15}$  B's age = 4 years.

∴ B's age = 30 years, A's age = 20 years.

An algebraic solution was furnished by Charles C. Cross.

Solutions of problems 98 and 99 were received from J. K. Ellwood, and solutions of problems 100 and 101 were received from J. Scheffer too late for credit in last issue.

#### ALGEBRA.

89. Proposed by G. A. MILLER, Ph. D., Instructor in Mathematics, Cornell University, Ithaca, New York.

Solve by quadratics,  $x^2 + y = 7$  . . . . (1),  
 $x + y^2 = 11$  . . . . . (2).

I. Solutions by W. F. BRADBURY, A. M., Head Master Cambridge Latin School, Cambridge, Mass.

The late Professor Quimby, of Dartmouth College, pronounced equations of this form insoluble except by Descarte's formula for bi-quadratics.

Various ways of getting these values of  $x$  and  $y$  in these particular equations I have seen.

(a) If  $x$  and  $y$  are integral and positive, from (1) we know  $x^2$  must be 1 or 4; and from (2) we see that  $x^2$  cannot be 1. Hence  $x^2=4$ , and  $y=2$  (not  $\pm 2$ ). Hence  $y=3$ .

(b) Adding (1) and (2), multiplied by 4, and adding 2 to both members, we have  $4x^2+4x+1+4y^2+4y+1=74$ , or  $(2x+1)^2+(2y+1)^2=74$ .

Now if  $x$  and  $y$  are both integral then 74 is the sum of two perfect squares; the only perfect squares whose sum is 74 are 25 and 49, and as  $y > x$  it follows that  $(2x+1)^2=25$ , and  $(2y+1)^2=49$ , or  $x=2$ , and  $y=3$ . The answers obtained from calling  $2x+1=-5$  and  $2y+1=-7$  do not prove.

(c) Multiply (1) by  $x$ , then  $xy^2+x^2=11x\dots\dots(3)$ . From (3) subtract (2),  $xy^2-y=11x-7\dots\dots(4)$ . To (4) add twice (1),  $xy^2+2y^2-y+2x=11x+15\dots\dots(5)$ , or  $(x+2)y^2-y=9x+15$ , or  $y=3$ , or  $(-3x-5)/(x+2)$ , and  $x=-2$ , or  $x^2=7+(3x+5)/(x+2)$ .

This  $x^2=7+(3x+5)/(x+2)$  is a cubic equation which gives three other values for  $x$ .

II. Solutions by M. E. GRABER, Student in Heidelberg University, Tiffin, O.; W. F. BRADBURY, A. M., Cambridge, Mass.; and P. S. BERG, A. M., Principal of Schools, Larimore, N. D.

Equating values of  $x^2$  from (1) and (2),  $7-y=(11-y^2)^2\dots\dots(3)$ ,  $y^4-22y^2+y+114=0\dots\dots(4)$ .

Factoring  $(y-3)(y^3-3y^2-13y+38)=0$ .

$\therefore y-3=0$  and  $y=3$ , and  $x=2$ . The three other values of  $y$  can be found by reducing  $y^3+3y^2-13y+38=0$ . GRABER, BRADBURY, BERG.

Let  $z=y+1$ , and substitute for  $y$  in the latter disjunctive equation  $y^3+3y^2-13y+38=0$ , and we get  $z^3-16z-23=0$ . Assume  $z=\omega\sqrt[3]{r_1}+\omega^2\sqrt[3]{r_2}$ .

$\therefore r_1+r_2=23$ , and  $\sqrt[3]{r_1r_2}=\frac{1}{3}$ .  $\therefore r_1r_2=\frac{4}{27}$ .

$\therefore r=\frac{1}{2}[23\pm\sqrt{(23^2-\frac{16}{27})}]=\frac{1}{2}[23\pm\frac{1}{9}\sqrt{(6303)}]$ .

$\therefore y=3$  or  $-1+\omega\sqrt[3]{[11\frac{1}{2}+\frac{1}{18}\sqrt{(6303)}]}+\omega^2\sqrt[3]{[11\frac{1}{2}-\frac{1}{18}\sqrt{(6303)}]}$  and  $x=2$  or  $\sqrt{[11-[-1+\omega\sqrt[3]{(11\frac{1}{2}+\frac{1}{18}\sqrt{(6303)}]}+\omega^2\sqrt[3]{(11\frac{1}{2}-\frac{1}{18}\sqrt{(6303)})}]}$ .

P. S. BERG.

III. Solution by A. H. BELL, Hillsboro, Ill., and W. F. BRADBURY, A. M. Cambridge, Mass.

Write (1)  $x^2-4=3-y\dots\dots(1)$ .

Write (2)  $9-y^2=x-2\dots\dots(2)$ .

(1)  $\times$  (2), etc.,  $(x^2-4)(9-y^2)-(x-2)(3-y)=0\dots\dots(3)$ .

$\therefore$  We have factors  $x-2=0$ ,  $x=2$ ;  $3-y=0$ ,  $y=3$ .

The other values of  $x$  and  $y$  can be easily found. None of these methods can be applied to  $x^2+y^2=a$  and  $x+y^2=b$ .

IV. Solution by SYLVESTER ROBINS, North Branch Depot, N. J.

Every quantity which is twice a square, as 2, 8, 18, 32, 50, 72, 98, 128,

162, 200, etc., is the sum of two consecutive numbers plus their squares. This is readily seen from  $a^2 + (a+1)^2 + a + (a+1) = 2a^2 + 4a + 2 = 2(a+1)^2$ .

Consequently in  $x^2 + y + x + y^2 = 7 + 11 = 18$ , the greater of the two unknowns is  $\sqrt{[\frac{1}{2}(18)]} = 3$ , and the less is a unit smaller  $= 2$ .

V. Solution by COOPER D. SCHMITT, A. M., Professor of Mathematics, University of Tennessee, Knoxville, Tenn.

(a) Adding (1) and (2),  $x^2 + x + y^2 + y = 18$ , or  $x^2 + x + \frac{1}{4} + y^2 + y + \frac{1}{4} = 18 + \frac{1}{4}$ .  $(x + \frac{1}{2})^2 + (y + \frac{1}{2})^2 = 7\frac{1}{4} = \frac{29}{4} + \frac{49}{4}$ .

Whence  $x + \frac{1}{2} = \frac{5}{2}$  or  $\frac{7}{2}$ ,  $y + \frac{1}{2} = \frac{3}{2}$  or  $\frac{5}{2}$ , or  $x = 2$ ,  $y = 3$ , the other answer not being applicable.

(b) From (1),  $y - 3 = 4 - x^2$ ; from (2),  $y^2 - 9 = 2 - x$ , or  $y - 3 = \frac{2-x}{y+3}$ .

Whence  $\frac{2-x}{y+3} = 4 - x^2$ , or  $x^2 - \frac{x}{y+3} = 4 - \frac{2}{y+3}$ , or  $x^2 - \frac{x}{y+3} + \frac{1}{4(y+3)^2} = 4 -$

$$\frac{2}{y+3} + \frac{1}{4(y+3)^2}. \quad x - \frac{1}{2(y+3)} = 2 - \frac{1}{2(x+3)},$$

whence  $x = 2$ , and hence  $y = 3$ .

(c) From (1),  $y = 7 - 2$ .

Substituting in (2),  $x + 49 - 14x^2 + x^4 = 11$ , or  $x^4 - 14x^2 + x + 38 = 0$ .

Factoring  $(x-2)(x^3 + 2x^2 - 10x - 19) = 0$ ,

whence  $x = 2$  for one root.

The cubic  $x^3 + 2x^2 - 10x - 19$  have 3 real roots, two negative and one positive. The positive root is between 3 and 4; the negative roots between  $-1$  and  $-2$ , and  $-3$  and  $-4$ .

These are found by Horner's method to be as follows:

$$x_1 = 3.13+, \quad x_2 = -1.74+, \quad x_3 = -3.27+.$$

From these the corresponding value of  $y$  can be found.

For a different solution see *Fisher and Schwatt's Algebra*, page 576.

VI. Solution by G. H. RICHARDS, Hillsboro, Ill.

$$(2) - (1) \quad y^2 - x^2 = 4 + y - x = 1(4 + y - x), \quad (y+x)(y-x) = 1(4 + y - x) \dots (3).$$

$$\therefore y - x = 1, \quad y + x = 4 + y - x \dots (4, 5).$$

$$(4) + (5) \text{ gives } 2y = 5 + y - x. \quad y = 5 - x.$$

$$\therefore x^2 - x = 2. \quad \therefore x = 2, \quad y = 3.$$

VII. Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa.

Let  $t = 3 - y$ . This value of  $3 - y$  in (1) and (2) gives

$$x^2 - 4 = t \dots (3), \quad x - 2 = t(3 + y) \dots (4).$$

$$\therefore x^2 - 4 = \frac{x-2}{3+y}. \quad \therefore x^2 - \frac{x}{3+y} = 4 - \frac{2}{3+y}.$$

$$\therefore x^2 - \frac{x}{3+y} + \frac{1}{4(3+y)^2} = 4 - \frac{2}{3+y} + \frac{1}{4(3+y)^2}.$$

$$\therefore x - \frac{1}{2(3+y)} = 2 - \frac{1}{2(3+y)}.$$

$$\therefore x=2, y=3.$$

From (1),  $y=7-x^2$ . This in (2) gives  $x^4-14x^2+x+38=0$ .

$$\therefore x=2 \quad y=3$$

$$x=3.13132 \quad y=-2.80516$$

$$x=-3.28316 \quad y=-3.77914$$

$$x=-1.84816 \quad y=3.5843$$

$$x^4-14x^2+x+38=(x^2-5\frac{1}{9}x+6\frac{2}{9})(x^2+5\frac{1}{9}x+6\frac{6}{9}) \text{ approximately.}$$

$$\therefore x^2-5\frac{1}{9}x+6\frac{2}{9}=0. \quad x^2+5\frac{1}{9}x+6\frac{6}{9}=0.$$

$$\therefore x=2 \quad y=3$$

$$x=3\frac{1}{9}=3.1313 \quad y=-2.8052$$

$$x=-3.2832 \quad y=-3.7792$$

$$x=-1.8481 \quad y=3.5843$$

Also solved by *W. H. DRANE*.

VIII. Solution by *J. M. BOORMAN*, Consultative Mechanician and Counselor at Law, Woodmere, Long Island, N. Y.

"Empirical" and "Practical" (note page 174, problem 83) each find all the roots by quadratics. "Empirical" (page 173) is here ( $fx=x$ ).

$$\therefore \Sigma^4-2(11\pm 1/\sqrt{114})\Sigma^2=\frac{1}{4}(1/\sqrt{114}), \sqrt{114}=\dots\dots\dots 10.677078, 252031.$$

$$\begin{aligned} \text{Complete the square by squaring } (11+1/\sqrt{114}), \text{ which squared} \\ =125+22/\sqrt{114}=469.895721, 544682 \end{aligned}$$

and adding to both sides

$$\text{We have } \Sigma^4-2[11+1/(11^2-7)]\Sigma^2+469.8957, \text{ etc.,} =\dots\dots\dots 469.919136, 189971$$

$$\text{since } \frac{1}{4}(1/\sqrt{114})=\dots\dots\dots 0.023414, 645289$$

$$\begin{aligned} \text{Extracting square roots, } \Sigma^2=\dots\dots\dots 21.677078, 252031+21.677618, 323745 \\ =43.354696, 575776 \text{ and } \Sigma=\pm 6.584428, 340849\dots(3). \end{aligned}$$

$$\text{True } \Sigma=6.584428, 340330, 491744, 944338, 239380, 16, \text{ etc.}\dots(A).$$

$$\text{Error}=0.000000, 000518+.$$

To save labor, as (2) transposes to  $y^2-9=2-x$ , therefore  $y-3=[2/(y+3)]-[x(y+3)]$ .  $\therefore y=3+[2(y+3)]-[x(y+3)]\dots(B)$ . By (B) in (1)  $x^2-[1/(y+3)]x+\square, =7-3-[1/(y+3)]2+\square, =4-[1(y+3)]2+\square, \dots(C)$ . Extracting the square root and solving for  $x$ ,  $x=\frac{1}{2}\{[1(y+3)]\pm_1[1/(y+3)]\}\mp(-2)\dots(D)$ !

$\therefore x=[1/(y+3)]-2\dots(E)$ ;  $x_1=2\dots(F)$ . By (F) in (1)  $y_1=3$ ,  $\therefore 3$  from (A) is  $y_a=3.584428, 340330, 491744, 944338, 239380, 161894, 6\pm$

and  $x_a=-1.848126, 526964, 403553, 538300, 209036, 266178, 2\mp$   
 $x_1+x_a=S=(1/\Sigma)=0.151873, 47+\dots(4)$  and  $(4)-2=-1.848126, 5+$ .

(E) in (1) gives  $y^2+[1(y+3)]=13$ , whence  $y^3+3y^2-13y=38\dots(G)$  has the three non-obvious roots.  $\therefore x^3+2x^2-10x=19\dots(H)$  by parity. But  $y_a$  and  $x_a$  are known by  $\Sigma$  and  $S$ . Therefore easily by quadratics:

$$\begin{aligned}
 x &= 3.131312, 518250, 572965, 804300, 733409, 211483, 7 \pm \\
 y &= -2.805118, 086952, 744853, 053572, 398087, 397376, 0 \mp \\
 x_{11} &= -3.283185, 991286, 169412, 266000, 514372, 745305, 5 \pm \\
 y_{11} &= -3.779310, 253377, 746891, 890765, 841292, 764578, 6 \mp
 \end{aligned}$$

\* $S^3 \mp 1, 2xS^2 - 2yS = \mp f \dots (J)$  by  $y = \mp 1, Sx + \frac{1}{2}(S^2 \pm 1, \Sigma) \dots (K)$  in  $(J)$ .

$\therefore x^2 \mp 1, Sx = 7 - \frac{1}{2}(S^2 \pm 1, \Sigma)$  give by quadratics all  $x$ 's and  $y$ 's with no further aid.

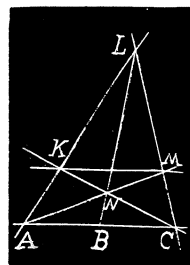
## GEOMETRY.

102. Proposed by B. F. FINKEL, A. M., M. Sc., Professor of Mathematics and Physics, Drury College, Springfield, Mo.

Ohne Benutzung des Cirkels eine Strecke  $AC$  zu halbiren, wenn eine Parallele der Geraden  $AC$  gegeben ist. [Reye's Geometrie der Lage, Part I, page 191].

I. Solution by C. HORNING, Professor of Mathematics, Heidelberg University, Tiffin, Ohio.

Wir ziehen durch irgend einen Punkt  $L$  ausserhalb der gegebenen Parallele die Linien  $AL$  und  $CL$  welche die Parallele in resp.  $K$  und  $M$  schneiden. Dann ziehen wir die Linien  $AM$  und  $CK$  die sich im Punkte  $N$  schneiden. So bestimmt die Diagonale  $LN$  des Vierecks  $KLMN$  den gesuchten Halbierungspunkt  $B$  der Strecke  $AC$ . Denn der vierte von  $B$  harmonisch getrennte Punkt, in welchem die zweite Diagonale  $KM$  des Vierecks die Gerade  $ABC$  schneidet, liegt uendlich fern.



II. Solution by the PROPOSER.

GIVEN the line  $AC$  and the line  $KM$  parallel to it,  
To BISECT  $AC$  without using the circle.

CONSTRUCTION. 1. Through  $A$  draw any two lines intersecting the parallel line in the points  $K$  and  $M$ .

2. Draw the lines  $CK$  and  $CM$ ,  $CK$  intersecting  $AM$  in  $N$  and  $CM$  intersecting  $AK$  in  $L$ .

3. Draw the diagonal,  $LN$ , of the 4-side,  $LMNK$ , and its intersection  $B$  with  $AC$  will be the required point of bisection.

The points  $A$ ,  $B$ ,  $C$ , and the point at infinity, *i. e.* the intersection of  $AC$  and  $MK$ , constitute a harmonic range of points and the point at infinity is harmonically separated from  $B$  by the points  $A$  and  $C$ .

This problem was also solved by G. B. M. ZERR, C. C. CROSS, J. SCHEFFER, A. F. KOVARIK, M. A. GRUBER, NELSON L. RORAY, ELMER SCHUYLER, and JOHN MANIE.

103. Proposed by FREMONT CRANE, Sand Coulee, Mont.

A horse is tethered with a rope which is attached to a stake  $B$  on the edge of a circular pond containing one acre. How long must the rope be to allow the horse to graze over one acre? [From *Home Study Magazine*, problem 249].